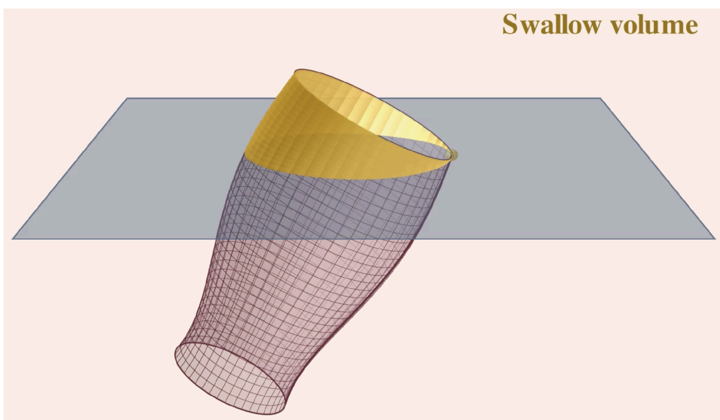
INTERNAL ASSESSMENT

Mathematics: Analysis and Approaches

Splitting the G

Finding the perfect angle to drink a pint of Guinness

using volumes of revolution and a tilted-plane model



A mathematical exploration

By James Fenwick

Introduction and rationale

Mathematics, my teachers keep insisting, is everywhere. For years I nodded politely and assumed they meant bridges and bank accounts. Then, one evening in a pub, I watched a friend raise a pint of Guinness **(non alcoholic ofcourse)**, take a single confident gulp, lower the glass, and stare in genuine reverence at the result. The dark line of the stout had landed exactly through the middle of the *G* on the glass. He had, in the language of the pub, *split the G*. The table erupted. I, however, had quietly stopped listening to the celebration, because a much more interesting question had appeared: was that a fluke, or is there a correct angle, one that works every time?

"Splitting the G" is a Guinness drinking challenge that went viral on TikTok and Instagram. The rules are simple: with your *first* sip of a freshly poured pint, you must drink exactly enough that the boundary between the stout and the foam settles halfway down the letter *G* in the Guinness logo. A millimetre either way and you have failed; the social consequences are severe. As a challenge it is part skill, part showmanship, and, I suspected, entirely solvable with the volume-of-revolution integration we had just met in class.

Aim and approach

The aim of this exploration is to determine the **angle from the vertical at which a full pint glass should be tilted** so that exactly the right volume of Guinness pours out to split the G. My plan has four stages:

1. Model the curved profile of a real pint glass as a function $r(y)$, the radius at height y.
2. Use the volume of revolution to find the total capacity of the glass, and the *target volume* — the Guinness sitting above the middle of the G, which must be removed.
3. Model tilting the glass as slicing the solid with a *tilted plane*, and find the poured ("swallow") volume as a function of tilt angle.
4. Solve for the angle at which the swallow volume equals the target, then check the prediction against footage of a real attempt.

Because I wanted the model to correspond to a real object, I had intended to collect my own primary data by photographing and measuring an official Guinness glass, rather than relying on idealised dimensions. Unfortunately al the pubs I frequent are dark and dingy and unsuitable for photography. The photos I did take were unsuitable and without the proper perspective. Ideally I would have used a tripod to reduce blur and a long subject distance to reduce perspective error, but I was too shy to ask the other patrons to vacant their seat so I could set up my tripod. Instead I have to rely on stock footage of an unknown but assumed large subject distance. The published specification of the glass is a capacity of **568 mL** (one imperial pint), a height of roughly 15 cm and a maximum diameter of about 8.5 cm — useful figures to check my model against.

Stage 1 — Modelling the shape of the glass

Collecting the data

A pint glass is a solid of revolution: every horizontal cross-section is a circle, but the radius changes with height in that distinctive "tulip" curve. To capture the shape I selected a photo of empty glass straight-on, imported the photograph into *Desmos*, and calibrated the image so that 1 unit on the axes equalled 1 cm, with the base of the cavity at the origin and the central axis lying along the y-axis. It turns out plotting the shape of an empty glass is quite difficult due to the refraction of light so I switch to a full pint glass, where I could see the shape better.

Figure 1. The official Guinness glass used. The left glass shows the target: stout and foam splitting the G.

I then traced the right-hand edge of the glass cavity by placing a table of points (x_1, y_1) directly onto the photograph, working upward from the base to the rim in roughly 1 cm steps. Each point gives the radius of the glass at that height.

Figure 2. Tracing the cavity edge in Desmos, with the photograph calibrated to centimetres.

Fitting a function to the edge

The tulip shape, narrow base, bulging belly, slight narrowing at the rim, is not monotonic, so a straight line or a cone would be hopeless, A polynomial of degree four can bend twice, which matches the silhouette, so I fitted a quartic regression to my traced points:

$$r(y) = a\,y^4 + b\,y^3 + c\,y^2 + d\,y + k$$

Desmos returned the coefficients below, with a coefficient of determination of $R^2 = 0.989$ — an excellent fit for a hand-traced curve.

a	0.000259
b	−0.01079
c	0.13946
d	−0.51374
k	3.30082

Plotting $x = r(y)$ laid the curve neatly along the edge of the photographed glass, confirming the fit visually as well as numerically.

Figure 3. The fitted quartic r(y) (blue) overlaid on the traced points and the glass.

> **Reflection.** *A quartic is a deliberate compromise. A higher-degree polynomial would hug my points more closely, but it would start fitting the wobble in my hand-placed dots rather than the true shape of the glass, so I stopped at degree four, where the curve is smooth and the residuals are small.*

Stage 2 — The volume of the glass

With $r(y)$ in hand, I can find the volume by the **disc method**. Imagine slicing the glass horizontally into a stack of extremely thin discs, like a tower of coins. A disc at height y is a circle of radius $r(y)$, so its area is $\pi\ r(y)^2$, and if its thickness is the tiny amount dy its volume is $\pi\ r(y)^2\ dy$. Summing infinitely many such discs is precisely a definite integral:

$$V = \int \pi \cdot r(y)^2\ dy \quad (\text{from } y = y_{base} \text{ to } y = y_{top})$$

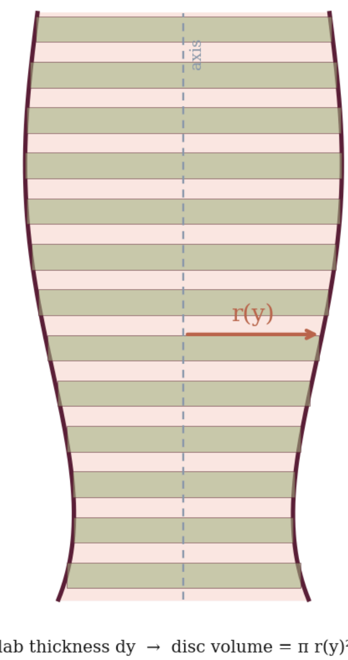


Figure 4. The disc (volume-of-revolution) method: the glass as a stack of thin circular slabs.

Evaluating this from the base ($y = 0.4$) to the rim ($y = 15$) in Desmos gives:

$$V_{full} = \mathbf{547.6\ mL}$$

This is satisfyingly close to the official 568 mL, with a percentage error of about 3.6%. Although I am confident in the shape of my curve and height of my pint class, I am not so confident in the scale of my photo, by adjusting k just a tiny bit I can make the radius of my pint glass bigger to match the figure of 568ml.

Stage 3 — The volume that must be removed

To split the G, the stout must end up halfway down the letter G. On my calibrated image the middle of the G sits at a height of $y_G = 9.63\ cm$. The Guinness that must leave the glass is therefore everything *above* that height — the same disc-stacking integral, just with a higher lower limit:

$$V_{target} = \int \pi \cdot r(y)^2\ dy \quad (\text{from } y = 9.63 \text{ to } y = 15) \quad = \mathbf{251.6\ mL}$$

That is about **46% of the entire pint** in a single gulp. When I first computed this I assumed I had made an error, nearly half a pint is an enormous mouthful. But the glass is tulip shaped and having a larger volume at the top can be expected when volume of each hypothetical vertical disc is dependent on the radius squared.

> **Reflection.** *It is worth being explicit about an assumption here: I have treated the glass as full of liquid to the rim. A real pint has a foam head occupying the top centimetre or two, and the split-the-G line is really the stout–foam boundary, not the top of the liquid. Modelling the head as ordinary liquid slightly over-states the volume removed. I revisit this in the evaluation.*

Stage 4 — The tilted-plane ("swallow") model

Now the interesting part: how do I physically remove exactly 251.6 mL? I tilt the glass and drink. The crucial physical insight is that **a liquid surface always stays horizontal**. When I tip the glass, the Guinness does not tilt with it — the surface stays level while the glass leans. So, viewed in the frame of the glass, the surface becomes a *flat plane that pivots about the point on the rim* where the stout pours over my lip. Everything above that plane leaves the glass; everything below stays. I think of it as the *swallow volume*.

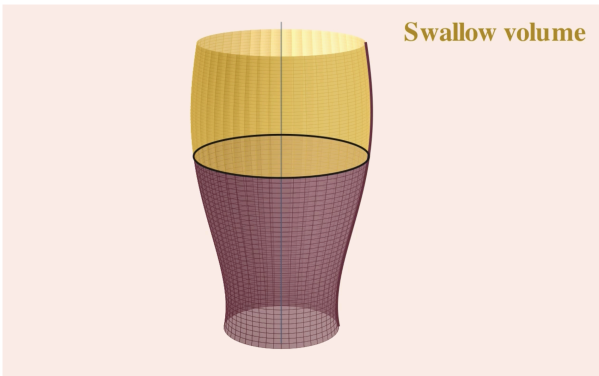


Figure 5. The solid of revolution, with the horizontal cut showing the volume above the G (yellow) that must be swallowed.

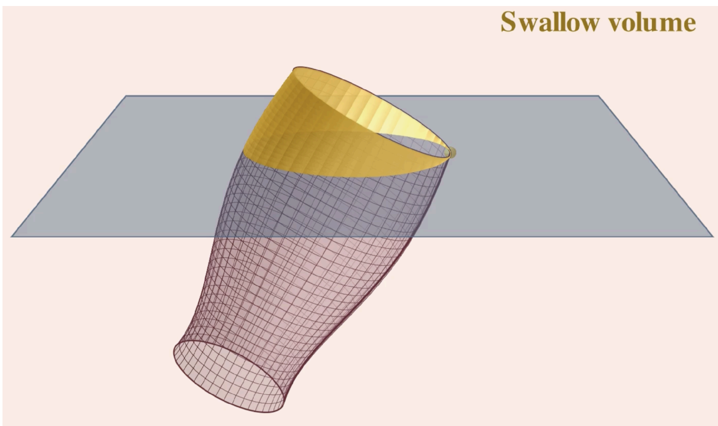


Figure 6. Tilting the glass: the liquid surface is modelled as a plane pivoting on the rim. The yellow region above the plane is what pours out.

Setting up the geometry

Let θ be the angle of the glass from the vertical (so $\theta = 0$ is upright). Let $R = r(15)$ be the radius at the rim, where the pivot sits. The tilted plane, written in the glass's frame, has height

$$y_{plane}(x) = 15 + \tan\theta \cdot (x - R)$$

A point spills if it lies above this plane. Rearranging, at each height y the plane cuts across the disc at the horizontal position

$$c(y) = R + (y - 15)/\tan\theta$$

Everything in that disc with $x < c(y)$ pours out. Two checks reassure me the setup is right: as $\theta \to 0$ the plane is flat and nothing spills; as $\theta \to 90°$ the glass is on its side and almost everything spills.

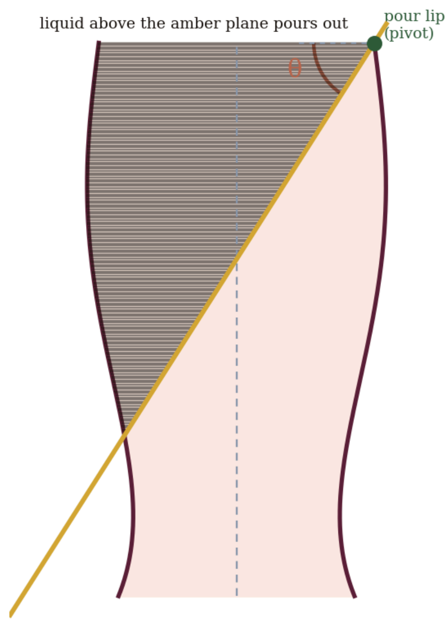


Figure 7. Cross-section of the tilted model. The amber line is the liquid plane, pivoting on the pour lip; θ is measured from the vertical axis.

Area of each sliced disc: the circular segment

Here is where the genuinely new mathematics enters. At a given height the plane slices the circular disc with a vertical line at $x = c(y)$. The part that spills is a **circular segment**, the crescent of a circle cut off by a chord. For a disc of radius $\rho = r(y)$ with the cut a (signed) distance c from the centre, the area of the spilling segment is

$$A(y) = \rho^2 \arccos(-c/\rho) + c\sqrt{(\rho^2 - c^2)}$$

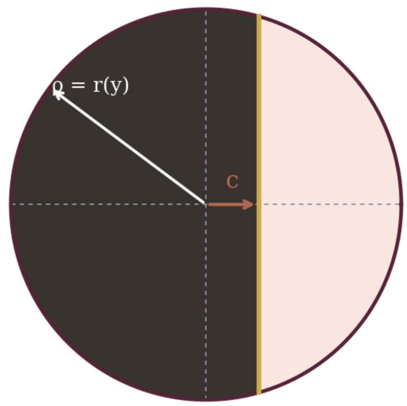


Figure 8. A single horizontal disc, sliced by the plane. The dark crescent (x < c) is the liquid that spills from that layer.

with two special cases: when the cut has passed the far edge of the disc ($c \geq \rho$) the whole disc spills and $A = \pi\rho^2$; when the cut has not yet reached the disc ($c \leq -\rho$) nothing spills and $A = 0$.

Summing the slices

Exactly as in Stage 2, I stack the sliced areas from base to rim. The total swallow volume, now a function of the tilt angle, is

$$V_{swallow}(\theta) = \int A(y)\, dy \quad \text{(from } y = 0.4 \text{ to } y = 15\text{)}$$

This integral cannot be solved neatly by hand — it has an $arccos$ of a function of y buried inside it, with θ tangled in as well. That is precisely the situation where a graphing calculator earns its keep, so I evaluated it numerically in Desmos with θ controlled by a slider.

Stage 5 — Solving for the angle

The problem now reduces to a single equation. I want the swallow volume to equal the target volume:

$$V_{swallow}(\theta) = V_{target} \quad \Longrightarrow \quad V_{swallow}(\theta) = 251.6 \text{ mL}$$

I plotted $V_{swallow}(\theta)$ against θ and looked for where it crosses the target line. The curve rises smoothly from 0 to the full 547.6 mL and is strictly increasing, so there is exactly one solution — a single perfect angle, as I had hoped.

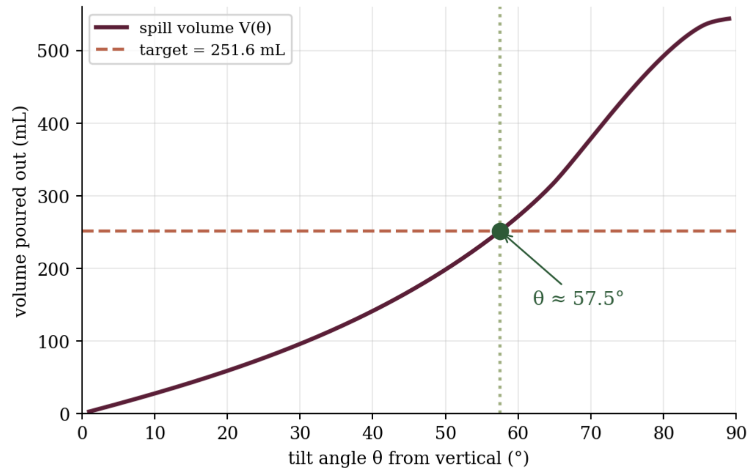


Figure 9. Swallow volume against tilt angle. The curve meets the target (251.6 mL) at one point only.

Reading off the intersection — and confirming it numerically — gives the headline result:

θ ≈ 57.5° from the vertical

In other words: tilt a full pint to about **57.5° from upright** (equivalently, about 32.5° above the horizontal), drink steadily until the stout stops flowing over the lip, set the glass down, and the line should land through the middle of the G. There it is — a pub legend reduced to a single number.

Stage 6 — Testing against reality

A prediction is only worth as much as its test. To check the model I analysed a still frame of a real, successful split-the-G attempt, drawing in the glass's central axis and a true vertical reference (from a plumb line in the background).

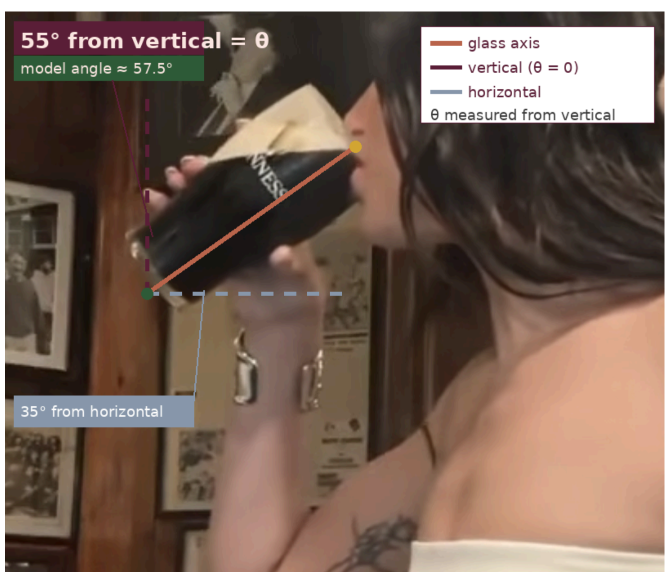


Figure 10. A real attempt (Dua Lipa- Youtube). The glass axis makes ≈ 55° with the vertical — strikingly close to the model's 57.5°.

The glass axis measures about **55° from the vertical**, within a couple of degrees of the predicted 57.5°. Given that the model is pure geometry and the photograph is a single hand-held frame, that agreement is better than I dared hope.

> **A note on how the angle is measured.** *I measure θ from the vertical because that is what the model uses: the upright glass is $\theta = 0$. It is tempting to measure the glass against the horizontal instead, but then I would read the complementary angle (≈35°) and it would no longer match the model directly. To avoid confusion I have drawn both reference lines on Figure 10 and labelled which is which.*

Two honest caveats. First, measuring an angle from one 2-D photograph is rough — perspective and an unseen tilt towards the camera could each shift the reading by several degrees. Second, the model predicts the *static* angle at which flow just stops; a real drinker has the glass moving, so the live tilt is a little past this. The match should be read as strong corroboration, not proof.

Analysis, limitations and extensions

Why the size of my glass did not matter

My modelled volume (547.6 mL) fell short of a true pint (568 mL), which initially worried me. But the final answer is an **angle**, and the angle depends only on the *ratio* of the target volume to the total volume — not on absolute size. If every length in the glass is scaled by a factor s, every volume scales by s^3, so the target and the total scale identically and the ratio is unchanged. Tilt angle is governed by **shape, not size**. This is why the prediction transfers cleanly from my slightly-too-small modelled glass to a real one — and why it should work on any standard pint glass of the same shape.

> **Reflection.** *This scale-invariance argument is the part of the exploration I am proudest of. It turns an apparent weakness (my volume being "wrong") into a justification for trusting the result, and it is a genuinely mathematical idea rather than a measurement fix.*

Limitations

- **The foam head.** I treated the glass as liquid to the rim. In reality the top of a pint is foam, and the split line is the stout–foam boundary, so the true volume of stout removed is a little less than 251.6 mL. Including the head would lower the predicted angle slightly.
- **A static model.** I assume the liquid settles to a flat plane and ignore the dynamics of an actual gulp — sloshing, momentum and the fact that nobody drinks in perfect quasi-equilibrium.
- **An idealised solid of revolution.** The model ignores glass wall thickness, the curve at the edge of the solid foot, all small, but real.
- **Where exactly is the G?** There is genuine disagreement about the target line, some aim for the middle of the G, others for the gap between the harp and the text. Since θ depends on y_G, this choice matters. Also I measure the g from a photo not from an actual glass, I suspect this may give another perspective error, if only I was brave enough to take my 30cm rule to the pub, if only my friends were kinder with their jibes

Extensions

Most Guinness pint glasses look the same, but they are not, both the Americans (473ml) and Europeans (500ml) measure their pints incorrectly, I would like to expand this investigation globally but I did not have the funds or spare time to go drinking abroad. I would also like to remodel the foam explicitly as a separate, less-dense layer, and, time and supervision permitting, to film myself attempting the split against a plumb line to gather repeated real-world data and test the prediction statistically.
Alternatively, I could attach a sexton to the top of a full pint glass, And Tip up to my desired angle and measure volume of the spillage.

Conclusion

Starting from nothing but a photograph of a pint glass and a viral pub challenge, I built a model that predicts a single, testable answer: tilt the glass to about **57.5° from the vertical** and you will split the G. The exploration drew together everything I find appealing about mathematics, fitting a function to messy real data, two distinct uses of integration (the volume of revolution and the summed circular segments), a numerical solve where algebra runs out, and a clean scaling argument that rescued the whole thing from my imperfect measurements. Best of all, when I checked the prediction against a real attempt, the maths and the pub agreed to within a couple of degrees.

My teachers were right, of course. The mathematics really was everywhere — it was hiding in the bottom of a pint glass the whole time. I look forward, purely in the name of empirical rigour, to collecting more data.

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